9.1.1 EXERCISES

For a link to all of the additional resources available for this section, click OSttS Chapter 9 materials.

In Exercises 1 - 13, write out the first four terms of the given sequence.

For help with these exercises, click on one or more of the resources below:

- Introduction to sequences
- Performing operations with factorials

1. $a_n = 2^n - 1,\ n \geq 0$
2. $d_j = (-1)^{\frac{j(j+1)}{2}},\ j \geq 1$
3. $\{5k-2\}_{k=1}^{\infty}$
4. $\left\{\dfrac{n^2+1}{n+1}\right\}_{n=0}^{\infty}$
5. $\left\{\dfrac{x^n}{n^2}\right\}_{n=1}^{\infty}$
6. $\left\{\dfrac{\ln(n)}{n}\right\}_{n=1}^{\infty}$
7. $a_1 = 3,\ a_{n+1} = a_n - 1,\ n \geq 1$
8. $d_0 = 12,\ d_m = \dfrac{d_{m-1}}{100},\ m \geq 1$
9. $b_1 = 2,\ b_{k+1} = 3b_k + 1,\ k \geq 1$
10. $c_0 = -2,\ c_j = \dfrac{c_{j-1}}{(j+1)(j+2)},\ m \geq 1$
11. $a_1 = 117,\ a_{n+1} = \dfrac{1}{a_n},\ n \geq 1$
12. $s_0 = 1,\ s_{n+1} = x^{n+1} + s_n,\ n \geq 0$
13. $F_0 = 1,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2},\ n \geq 2$ (This is the famous Fibonacci Sequence)

In Exercises 14 - 21 determine if the given sequence is arithmetic, geometric or neither. If it is arithmetic, find the common difference d; if it is geometric, find the common ratio r.

For help with these exercises, click on one or more of the resources below:

- Arithmetic sequences
- Geometric sequences

14. $\{3n-5\}_{n=1}^{\infty}$
15. $a_n = n^2 + 3n + 2$, $n \geq 1$
16. $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \ldots$
17. $\left\{3\left(\frac{1}{5}\right)^{n-1}\right\}_{n=1}^{\infty}$
18. $17, 5, -7, -19, \ldots$
19. $2, 22, 222, 2222, \ldots$
20. $0.9, 9, 90, 900, \ldots$
21. $a_n = \frac{n!}{2}$, $n \geq 0$.

In Exercises 22 - 30, find an explicit formula for the n^{th} term of the given sequence. Use the formulas in Equation 9.1 as needed.

For help with these exercises, click on one or more of the resources below:

- Finding the n^{th} term of a sequence
- Finding the n^{th} term of an arithmetic sequence
- Finding the n^{th} term of a geometric sequence

22. $3, 5, 7, 9, \ldots$
23. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$
24. $1, \frac{2}{3}, \frac{4}{5}, \frac{8}{7}, \ldots$
25. $1, \frac{2}{3}, \frac{1}{3}, \frac{4}{27}, \ldots$
26. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$
27. $x, -\frac{x^3}{3}, \frac{x^5}{5}, -\frac{x^7}{7}, \ldots$
28. $0.9, 0.99, 0.999, 0.9999, \ldots$
29. $27, 64, 125, 216, \ldots$
30. $1, 0, 1, 0, \ldots$
31. Find a sequence which is both arithmetic and geometric. (Hint: Start with $a_n = c$ for all n.)
32. Show that a geometric sequence can be transformed into an arithmetic sequence by taking the natural logarithm of the terms.
33. Thomas Robert Malthus is credited with saying, “The power of population is indefinitely greater than the power in the earth to produce subsistence for man. Population, when unchecked, increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will show the immensity of the first power in comparison with the second.” (See this webpage for more information.) Discuss this quote with your classmates from a sequences point of view.

34. This classic problem involving sequences shows the power of geometric sequences. Suppose that a wealthy benefactor agrees to give you one penny today and then double the amount she gives you each day for 30 days. So, for example, you get two pennies on the second day and four pennies on the third day. How many pennies do you get on the 30^{th} day? What is the total dollar value of the gift you have received?

35. Research the terms 'arithmetic mean' and 'geometric mean.' With the help of your classmates, show that a given term of a arithmetic sequence a_k, $k \geq 2$ is the arithmetic mean of the term immediately preceding, a_{k-1} it and immediately following it, a_{k+1}. State and prove an analogous result for geometric sequences.

36. Discuss with your classmates how the results of this section might change if we were to examine sequences of other mathematical things like complex numbers or matrices. Find an explicit formula for the n^{th} term of the sequence $i, -1, -i, 1, i, \ldots$. List out the first four terms of the matrix sequences we discussed in Exercise 8.3.1 in Section 8.3.

Checkpoint Quiz 9.1

1. Consider the sequence: $a_n = \dfrac{(-1)^n \, 3^{n-1}}{4^{n+1}}$, $n \geq 0$.

   (a) Write out the first five terms of the sequence.

   (b) Is this sequence arithmetic, geometric, or neither? Explain.

2. Consider the sequence: $a_1 = 10$ and $a_{n+1} = a_n + 2$ for $n \geq 1$.

   (a) Write out the first five terms of this sequence.

   (b) Is this sequence arithmetic, geometric, or neither? Explain.

   (c) Find an explicit formula for a_n.

3. Find an explicit formula for the sequence: $1, -\dfrac{2}{5}, \dfrac{4}{9}, -\dfrac{8}{13}, \dfrac{16}{17}, \ldots$

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2

9.1.2 Answers

1. $0, 1, 3, 7$
2. $-1, -1, 1, 1$
3. $3, 8, 13, 18$
4. $1, 1, \frac{5}{3}, \frac{5}{2}$
5. $x, \frac{x^2}{4}, \frac{x^3}{9}, \frac{x^4}{16}$
6. $0, \frac{\ln(2)}{2}, \frac{\ln(3)}{3}, \frac{\ln(4)}{4}$
7. $3, 2, 1, 0$
8. $12, 0.12, 0.0012, 0.000012$
9. $2, 7, 22, 67$
10. $-2, -\frac{1}{3}, -\frac{1}{36}, -\frac{1}{720}$
11. $117, \frac{1}{117}, 117, \frac{1}{117}$
12. $1, x+1, x^2+x+1, x^3+x^2+x+1$
13. $1, 1, 2, 3$
14. arithmetic, $d = 3$
15. neither
16. geometric, $r = \frac{1}{2}$
17. geometric, $r = \frac{1}{5}$
18. arithmetic, $d = -12$
19. neither
20. geometric, $r = 10$
21. neither
22. $a_n = 1 + 2n, \ n \geq 1$
23. $a_n = \left(-\frac{1}{2}\right)^{n-1}, \ n \geq 1$
24. $a_n = \frac{2^{n-1}}{2n-1}, \ n \geq 1$
25. $a_n = \frac{n}{3^{n-1}}, \ n \geq 1$
26. $a_n = \frac{1}{n^2}, \ n \geq 1$
27. $\frac{(-1)^{n-1}x^{2n-1}}{2n-1}, \ n \geq 1$
28. $a_n = \frac{10^n-1}{10^n}, \ n \geq 1$
29. $a_n = (n+2)^3, \ n \geq 1$
30. $a_n = \frac{1+(-1)^{n-1}}{2}, \ n \geq 1$